

## Errors, Accuracy, Precision, and Significant Figures

When presenting or reading about experimental results, the uncertainty of the result should be presented.

**Uncertainties** in experimental results are of two main types: those that result from fluctuations in measurements and those associated with the theoretical description of our result.

**Error:** the difference between a calculated or observed value and some “true” value.

Usually we cannot know what the true value is, and we can consider only *estimates* of the errors inherent in the experiment. If we repeat the experiment, the results may well differ from those of the first attempt. We can express this difference as a *discrepancy* between the two results. Discrepancies arise because we can determine a result only with a given *uncertainty*.

There are three general types of *error*, though only two are really important.

**Illegitimate errors:** involve problems with the way the experiment has been done. The only way to correct this is to start the experiment from the beginning.

**Systematic errors:** Reproducible errors that introduced by faulty equipment, calibration, or technique.

**Random errors:** Fluctuations in observations which yield differing results from experiment to experiment. This requires repeated experiments to reduce the effect of these errors.

These are closely related to *precision* and *accuracy*.

**Precision:** a measure of how carefully the result is determined without reference to the “true” value. The precision is also a measure of reproducibility of the result.

**Accuracy:** a measure of how close the result of the experiment is to the “true” value.

### **Questions:**

*So what kind of errors affects precision? Accuracy?*

It is obvious that we must consider the accuracy and precision simultaneously for any experiment. It would be a waste of time and energy to determine a result with high

precision if we knew it would be highly inaccurate. Conversely, a result cannot be considered to be extremely accurate if the precision is low. In general, when we quote the uncertainty in an experimental result, we are referring to the precision with which that result has been determined. The accuracy is typically represented in the measurement of a standard.

The precision of an experimental result is implied by the number of digits recorded in the results, although generally the uncertainty should be quoted specifically as well (this will be covered next week).

**Significant figures:** the number of digits recorded in a result that reflects the precision of the measurement. If no error is quoted, the last digit is the uncertain one.

**Rules for significant figures:**

1. The leftmost nonzero digit is the most significant digit.  
*0.0021 has two significant figures, while 0.4500 has 4 significant figures.*
2. If there is no decimal point, the rightmost nonzero digit is the least significant digit.  
*4500, 450, and 450000000 all have two significant figures.*
3. If there is a decimal point, the rightmost digit is the least significant digit, even if it is zero.  
*All of the following have three significant figures: 0.0312, 45.1, 400., and 40.0.*
4. All digits between the least and most significant digits are counted as significant digits.

For numbers that do not contain decimal points, the trailing zeros may or may not be significant. To clarify things, we use scientific notation (also known as exponential notation). Scientific notation uses the format  $\#.###$  (according to the number of significant figures)  $\times 10^y$ . Scientific notation allows us to express large or small numbers to the correct number of **significant figures** easily. For example, 400 cm would be express as  $4 \times 10^2$  for one significant figure,  $4.0 \times 10^2$  for two significant figures, or  $4.00 \times 10^2$  for three significant figures.

**Exercises:**

Determine the number of significant figures in the following measured quantities:

- (a) 478K                      (b) 6.010cm    (c) 0.0825g  
(d)  $6.022 \times 10^{23}$ atoms    (e) 7000ml      (f) 0.050mg

### **Rules for Significant Figures in Calculations:**

1. In addition and subtraction, the number of significant figures to the right of the decimal point in the final sum or difference is determined by the lowest number of significant figures to the right of the decimal point in any of the original numbers.

$$413.23 + 54.7 = 467.93 \text{ gets rounded off to } 467.9$$

$$321598.1 + .0111 = 321598.1111 \text{ gets rounded off to } 321598.1$$

2. In multiplication and division, the number of significant figures in the final product or quotient is determined by the original number that has the smallest number of significant figures.

$$2.8 * 4.5039 = 12.61092 \text{ gets rounded off to } 13$$

$$6.85/112.04 = 0.0611388789 \text{ gets rounded off to } 0.0611$$

3. Keep in mind that exact numbers obtained from definitions or by counting numbers of objects can be considered to have an infinite number of significant figures.

*If an object has mass 0.2786 g, then three such objects is  $0.2786\text{g} * 3 = 0.8358\text{g}$ , because the number 3 is actually 3.00000...by definition.*

### **Rules for rounding:**

1. If the fraction is greater than  $\frac{1}{2}$ , increment the new least significant digit.  
63.587 rounds to 63.59
2. If the fraction is less than  $\frac{1}{2}$ , do not increment.  
5.891 rounds to 5.89
3. If the fraction equals  $\frac{1}{2}$ , increment the least significant digit only if it is odd.  
6.325 rounds to 6.32, 6.315 rounds to 6.32

### **Exercises:**

Carry out the following arithmetic operations:

(a)  $65.336 + 47.893$

(b)  $32.4 - 0.128$

(c)  $65.3 * 0.065$

(d)  $6930 / 0.6975$

(e)  $45.65 + 32.63/8.000$