Notes 14

I. Modes of radioactive decay
II. Radioactive decay equation

β^- decay

$n \rightarrow p + \beta^-$, Z ⇑ (+1), N ⇓ (–1), A=constant (isobaric)

$$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + \beta^- + \bar{\upsilon}$$

β^- = beta particle or negatron

$\bar{\upsilon}$ = anti-neutrino

$$^{40}_{19}K \rightarrow {}^{40}_{20}Ca + \beta^- + \bar{\upsilon}$$

Q=1.312 MeV (energy equivalent to the difference in mass between reactants and products)

The transformation is accompanied by a discreet energy difference Q, yet the energy spectrum of the beta particles is continuous and the energy of a particular beta particle is < or equal to Q. For a particular decay, the difference in energy between Q and the kinetic energy of the beta particle is the energy of the anti-neutrino.

Kinetic energy of the beta particle + $\bar{\upsilon}$ = Q

Beta particle energies are typically tenths of an MeV to a little above an MeV.

$M_Z > M_{Z+1}$
-

β^+ (positron) decay

$p \rightarrow n + \beta^+$, $Z \Downarrow (-1)$, $N \Uparrow (+1)$, A=constant (isobaric)

$${}^{A}_{Z}P \rightarrow {}^{A}_{Z-1}D + \beta^{+} + \upsilon$$

β^+ = positron

υ = neutrino

$${}^{40}_{19}K \rightarrow {}^{40}_{18}Ar + \beta^{+} + \upsilon \qquad Q=0.49 + 2(0.51)\text{ MeV}$$

Positron decay requires a threshold energy of 1.02 MeV (to create the positron in the nucleus). The positron is unstable and after ejection from the nucleus reacts with an electron to produce two γ-rays of 0.51 MeV (the energy equivalent to the mass of an electron or positron) each.

The transformation is accompanied by a discreet energy difference, Q, yet the energy spectrum of the positrons is continuous and the energy of a particular positron is < or equal to Q minus 1.02 MeV. For a particular decay, the difference in energy between Q minus 1.02 MeV and the kinetic energy of the positron is the energy of the neutrino.

Kinetic energy of the positron + υ + 2(0.51 MeV) = Q

$M_Z > M_{Z-1} + 2 (M_e)$

-

Electron Capture

p + e $\rightarrow$ n , Z $\Downarrow$ (–1), N $\Uparrow$ (+1), A=constant (isobaric)

$$^{A}_{Z}P + e \rightarrow \; ^{A}_{Z-1}D + \upsilon$$

$$^{40}_{19}K + e \rightarrow \; ^{40}_{18}Ar + \upsilon \qquad Q=1.51\text{ MeV}$$

The net result of electron capture is similar to positron decay, but electron capture does not require a threshold energy.

The transformation is accompanied by a discreet energy difference, Q. As there are no other ejected particles, the neutrino is mono-energetic with an energy equal to Q.

Typically, the captured electron is from one of the inner shells. A higher energy electron from the outer shells will fill the position of the captured electron. This will generate an x-ray, with an energy equal to the difference in orbital energies. An even higher energy electron will fill the position of the electron that filled the captured electron position, generating another x-ray, etc. - the point being that electron capture is typically accompanied by x-rays (KeV energies).

-

$M_Z > M_{Z-1}$

-

γ – rays

Nuclear decays can leave the resulting nucleus in an excited (or metastable) state. That excited nucleus will decay to its ground state by emission of high energy electromagnetic radiation in the form of a γ – ray.

$${}^{40}_{19}K + e \rightarrow {}^{40}_{18}Ar^* + \upsilon \qquad Q=0.0442\ \text{MeV}$$

$${}^{40}_{18}Ar^* \rightarrow {}^{40}_{18}Ar + \gamma \qquad E_\gamma = 1.46\ \text{MeV (“isomeric transition”)}$$

Typical γ ray energies are tenths of an MeV to slightly more than an MeV

α – decay

α particle (2 neutrons and 2 protons) ejected from nucleus, Z ⇓ (–2), N ⇓ (–2), A ⇓ (–4)

$$^{A}_{Z}P \rightarrow \, ^{A-4}_{Z-2}D + \, ^{4}_{2}\alpha$$

α = alpha particle which is a ^{4}He nucleus

$$^{238}_{92}U \rightarrow \, ^{234}_{90}Th + \, ^{4}_{2}He \qquad Q=4.267 \text{ MeV}$$

$K.E._{\alpha} = 4.196$ MeV

$$Q = K.E._{\alpha} + K.E._{nucleus}$$

The transformation is accompanied by a discreet energy difference Q; Q is equal to the kinetic energy of the alpha particle and the recoiling nucleus. As both must have the same momenta, the alpha particles are mono-energetic. Alpha particle energies are typically 2 to 6 MeV. Only occurs in heavy nuclides (A>140), often followed by beta decay (uranium series)

$$M_{Z,N} > M_{Z-2,N-2} + M_{\alpha}$$

Spontaneous Fission

Nucleus splits into two nuclei plus neutrons

$$^{A}_{Z}P \rightarrow \, ^{A_1}_{Z_1}D + \, ^{A_2}_{Z_2}D + b\, ^{1}n + Q$$

$A_1 + A_2 + b = A$

$Z_1 + Z_2 = Z$

b is around 3

Q is around 200 MeV, mainly as kinetic energy of the fragments

$$^{238}_{92}U \rightarrow \, ^{136}_{52}Te + \, ^{99}_{40}Zr + 3n$$

One fissioning parent decays to a wide range of daughters (see figure). The daughters are beta-active. Because of the extremely large energy release, the kinetic energy of the daughters is large (fission-track dating). Only for A very large. Half-lives for spontaneous fission are about 10^{16} years. Released neutrons can react with nuclides that do not fission spontaneously to produce a nuclides that do fission spontaneously to produce a chain reaction.

Decay equation

Observation: $dN/dt \propto t$, where N is the number of atoms of a radioactive nuclide and t is time.

If we choose λ as the constant of proportionality and stipulate that is be positive, then we have the differential form of the radioactive decay equation:

$$dN/dt = -\lambda N$$

λ is called the “decay constant” and has units of t^{-1}. The rate of decay or “activity” is equal to λN. We can integrate this subject to the initial condition that $N=N_0$ at $t=0$:

$$dN/N = -\lambda\ dt$$

$$\ln N = -\lambda t + C$$

$$N = (e^{-\lambda t})\ (e^{C})$$

$$N_0 = 0 + e^{C}$$

$$N = N_0\ e^{-\lambda t}$$

We can calculate the “half-life = $t_{1/2}$”, which is the time required for half of the atoms of a radioactive nuclide to decay, as follows:

$$N/N_0 = 1/2 = e^{-\lambda t\ 1/2}$$

$$t_{1/2} = (\ln 2)/\lambda$$

We can calculate the "mean-life = τ", which is the mean time that an atom of a sample of a radioactive nuclide exists, as follows:

$$\tau = (1/N_0) \int_0^{N_0} t\, dN \qquad (dN = -\lambda N dt)$$

$$\tau = (1/N_0) \int_{\infty}^{0} t\, (-\lambda N dt)$$

$$\tau = (1/N_0) \int_0^{\infty} t\, (\lambda N dt) \quad (N = N_0\, e^{-\lambda t})$$

$$\tau = \int_0^{\infty} t\, (\lambda)(e^{-\lambda t})$$

$$\tau = -\left[((\lambda t + 1)/\lambda)(e^{-\lambda t})\right]_0^{\infty} = 1/\lambda$$

Substituting back into the integral form of the radioactive decay equation,

$$N_\tau = N_0\, e^{-\lambda\ (1/\lambda)}$$

$$N_\tau / N_0 = 1/e.$$

By substitution into the half-life equation:

$$t_{1/2} = (\ln 2)\ \tau$$